Stochastic Anytime Search in Autonomous Planning & Scheduling

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Goal: Develop the theory and algorithms for dynamic planning and scheduling under nonlinear discrete/procedural constraints.

Objectives:

- Formulate NASA planning problems as dynamic optimization under nonlinear discrete constraints.
- Develop formal theory and algorithms to solve dynamic optimization problems efficiently.

Key Innovations:

- Identify new dominance relations on nodes in dynamic optimization problems in discrete space.
- Extend necessary Euler-Lagrange conditions in optimal control theory in continuous space to counterpart conditions in discrete space and prove that they are necessary as well as sufficient.
- Extend *Aspen* to allow joint optimization of objective and constraints, based on a sound mathematical theory.

NASA Relevance:

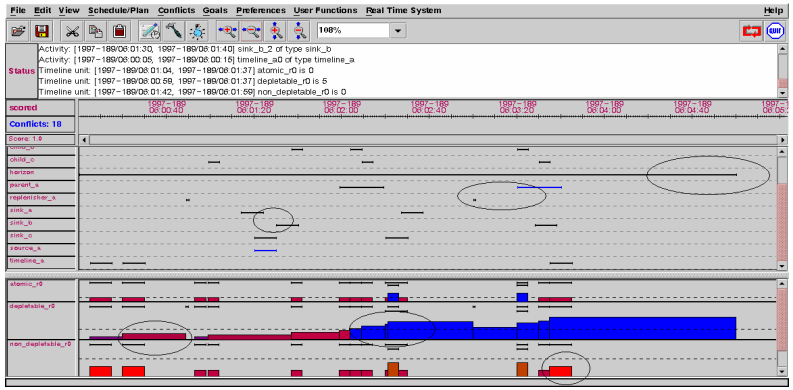
- Allow plug-and-play applications in NASA planners, like *Aspen*, to enable anytime planning and scheduling of time-critical NASA applications.

Accomplishments to date:

- Completed the theory of calculus of variations in discrete space.
- Applied the theory to demonstrate significant quality and time improvements on benchmarks as compared to those of *Aspen*.
- Completed a Web-based prototype for solving nonlinear discrete optimization problems.

Description/Schedule

- Theory of Lagrange multipliers for discrete constrained optimization: necessary and sufficient conditions for constrained local minima.
- Stochastic search algorithms: unified algorithm to look for discrete-neighborhood saddle points.
- Calculus of variations in discrete space:
  - New dominance relations on nodes in dynamic optimization;
  - Discrete-neighborhood Euler-Lagrange conditions (DNEL) proved using the theory of Lagrange multipliers for discrete optimization.
- Application of theory and stochastic search to *Aspen*.



Applying DNEL during iterative repair

Schedule:

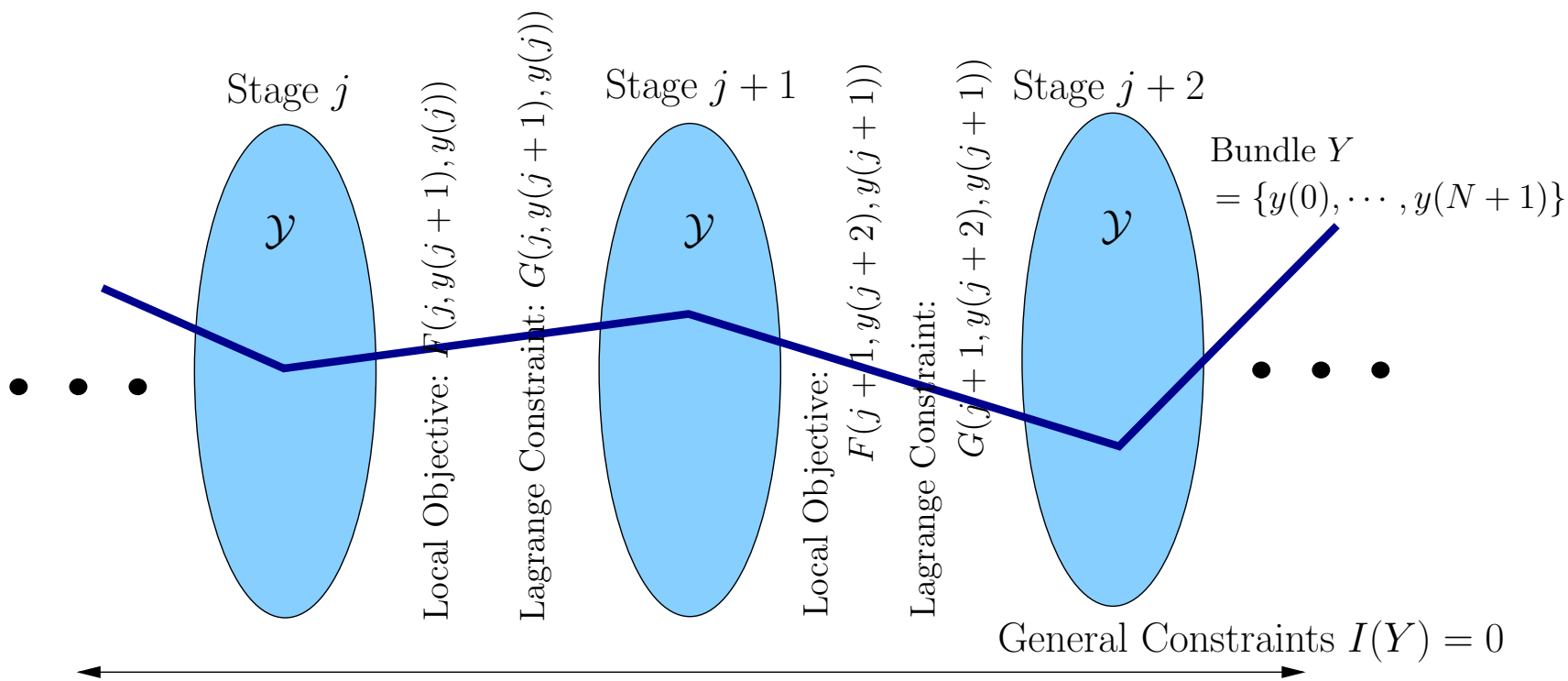
- 7/15/02: Demonstrate algorithms on benchmark applications in *Aspen*.
- 8/31/02: Complete integration of algorithms into *Aspen*.
- 11/31/02: Investigate applications in time-variant planning, multi-objective scheduling, and general planning problems.

Dynamic Optimization Problems

Optimization problems with time varying dynamic variables

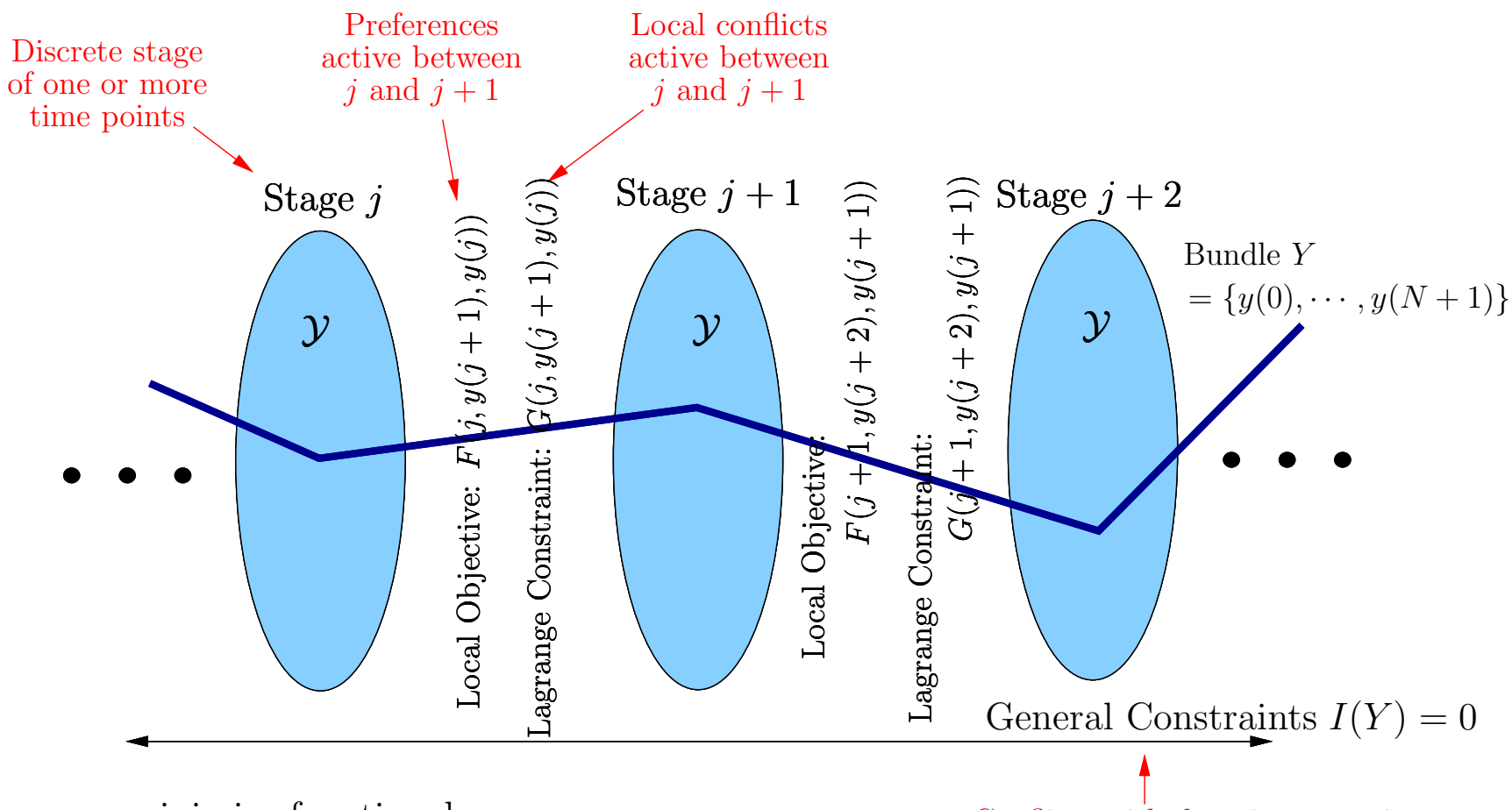
Space	Time: Continuous	Time: Discrete
Continuous	Control Theory and Classical Calculus of Variations	Control Theory and Classical Calculus of Variations
Discrete	No Previous Work (Potentially Interesting)	Current focus with functions that may not be continuous and differentiable (NP-hard)

Objectives and Constraints in Dynamic Optimization



$$\text{minimize functional } J[\{y(j)\}] = \sum_{j=0}^{N} F(j, y(j+1), y(j))$$

Dynamic Optimization Problems Modeled by Aspen

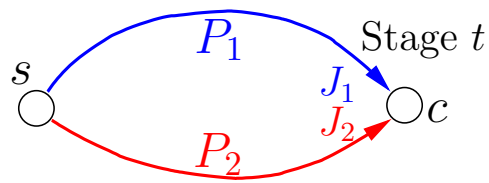


minimize functional

$$J[\{y(j)\}] = \sum_{j=0}^{N} F(j, y(j+1), y(j))$$

Path Dominance: Principle of Optimality

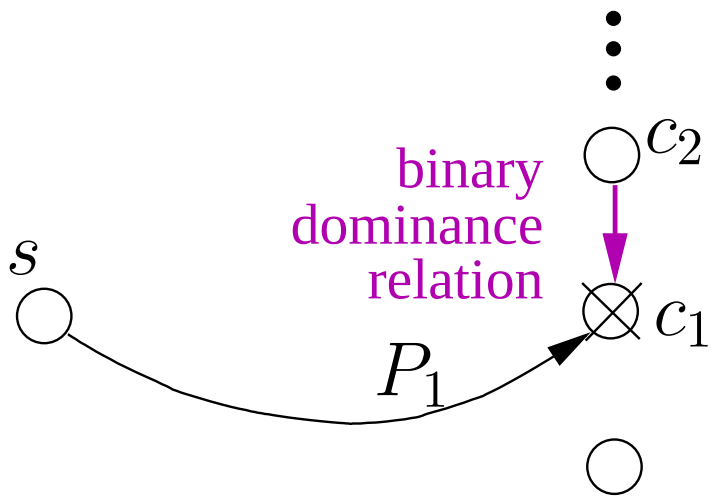
- Principle of Optimality in dynamic programming applied on feasible state c



If c lies on the optimal path between s and d and $J_2 \leq J_1 \implies P_2 \rightarrow P_1$

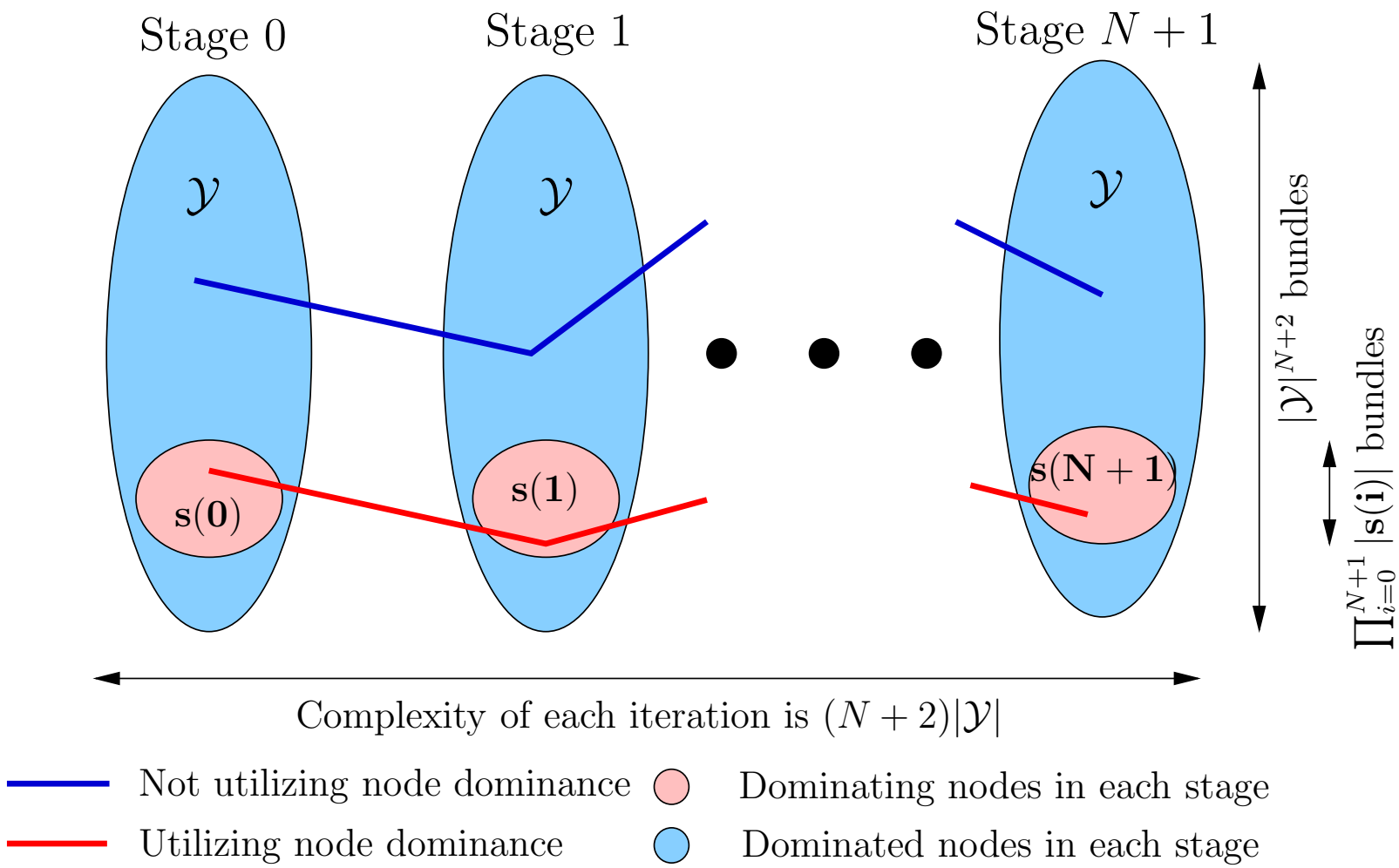
- Can only be applied to unconstrained problems or problems with local (Lagrange type) constraints
  - Polynomial worst-case complexity: $O\left(N|\mathcal{Y}|^2\right)$
- Not applicable to problems with general constraints $I(Y) = 0$
  - Exponential worst-case complexity: $O\left(|\mathcal{Y}|^N\right)$, assuming NP hard

Node Dominance



Termination of c_1 by
binary dominance relation
(not related to P_1)

Benefits of Using Node Dominance



Worst-Case Complexities in Path and Node Dominance

Constraint Type	Without General Constraints (Conventional DP)		With General Constraints (Without Path Dominance)	
Dominance Used	With Path Dominance	With Path and Node Dominance	Without Node Dominance	With Node Dominance
Complexity	$O\left(N\lvert\mathcal{Y}\rvert^2\right)$	$O\left(N\lvert\mathcal{Y}\rvert + \sum_{i=1}^{N-1} \lvert\mathbf{s}(\mathbf{i})\rvert \times \lvert\mathbf{s}(\mathbf{i}+\mathbf{1})\rvert\right)$	$O\left(\lvert\mathcal{Y}\rvert^N\right)$	$O\left(N\lvert\mathcal{Y}\rvert \prod_{i=1}^{N} \lvert\mathbf{s}(\mathbf{i})\rvert\right)$
Condition Type	Necessary and Sufficient		–	Necessary

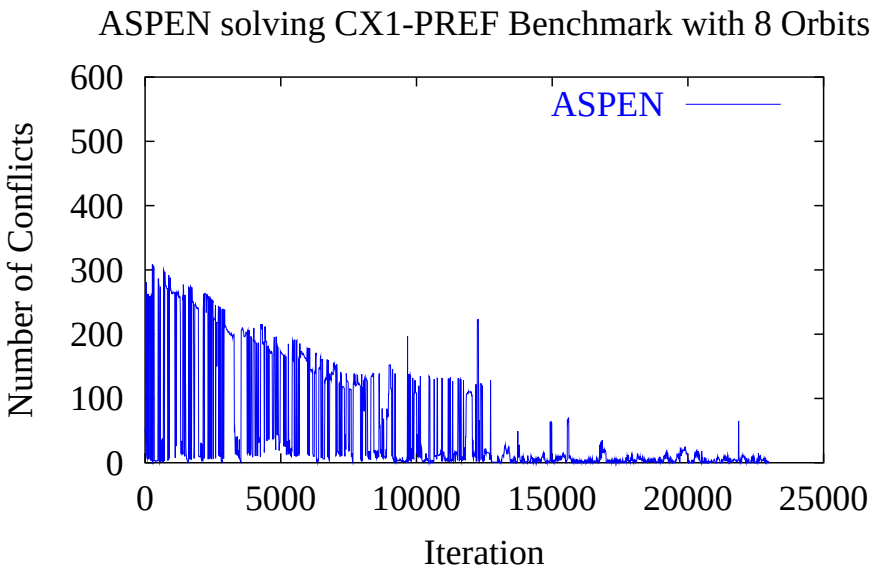
Node dominance works well when $\lvert\mathbf{s}(\mathbf{i})\rvert \ll \lvert\mathcal{Y}\rvert$

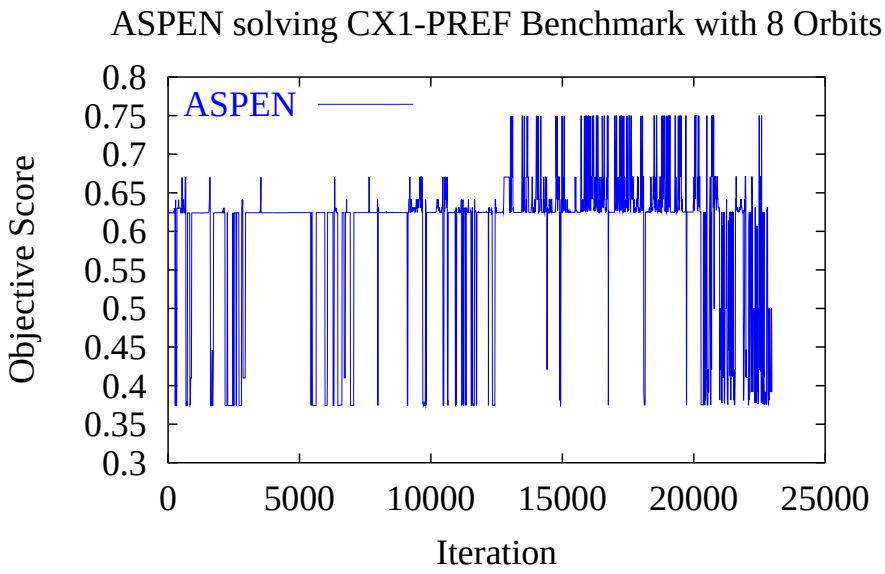
Dominance in ASPEN

- Path dominance (Principle of Optimality) cannot be applied
  - Planning problems have general constraints spanning across multiple or all stages
- Node dominance relations are open

Illustration of Conflicts/Scores vs Iterations in ASPEN Search

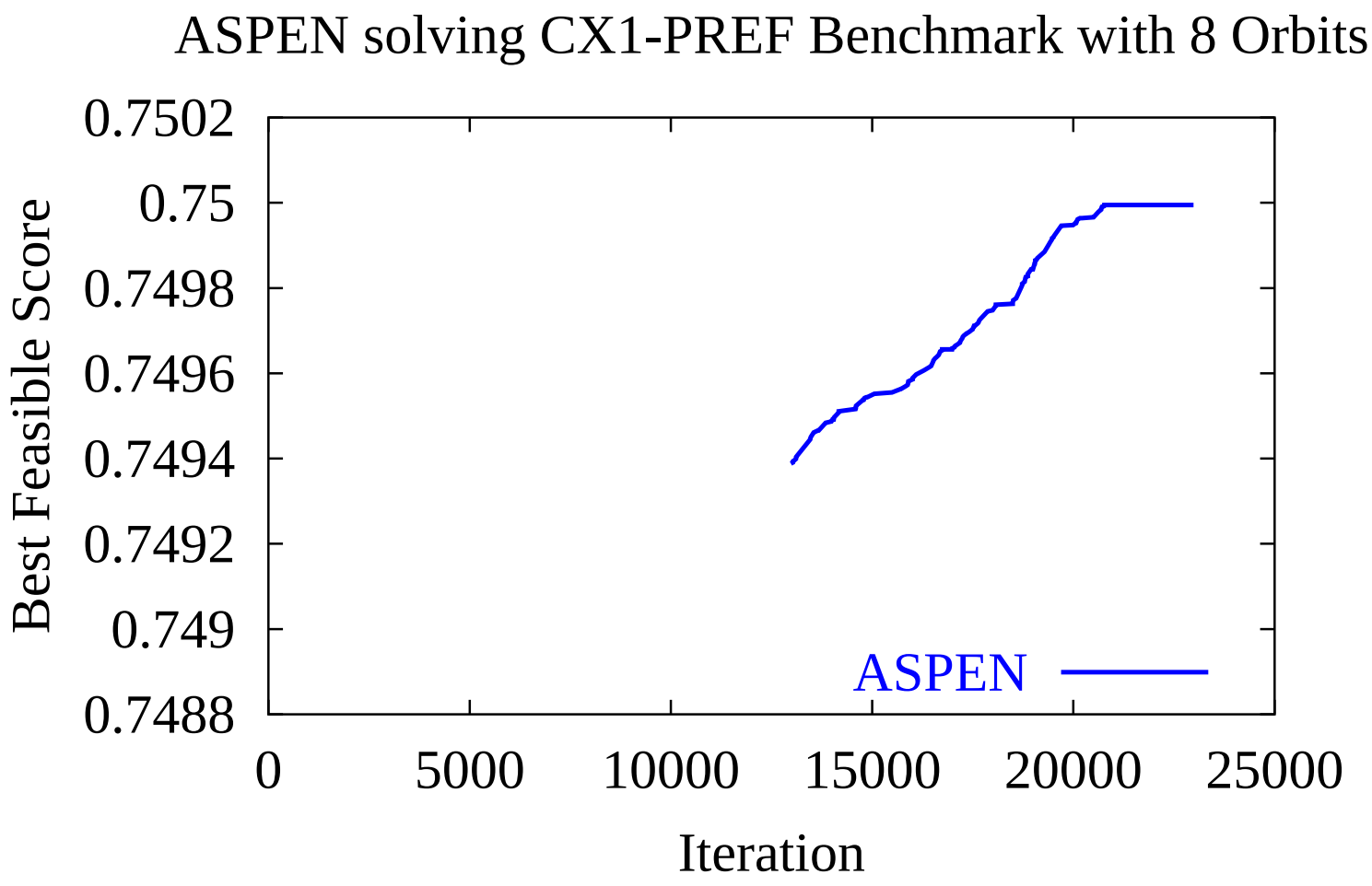
Repair strategy alternates between repairs of constraints
and optimization of objective





ASPEN solving CX1-PREF Benchmark with 8 orbits

Best Feasible Scores in an 8-Orbit Problem



Calculus of Variations in Discrete Space: General Idea

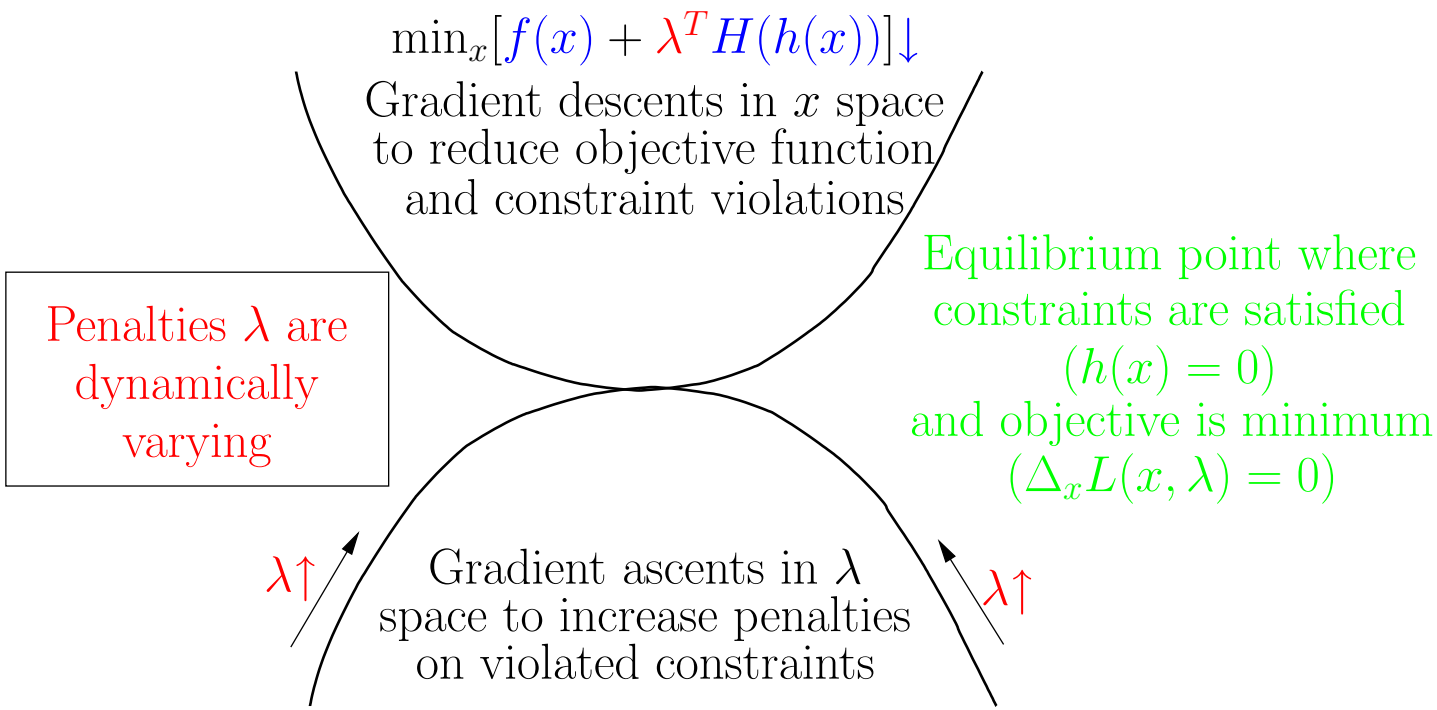
- Dynamic optimization is discrete constrained optimization
  - Locally optimal bundles $\equiv$ discrete-neighborhood saddle points SP_{dn}
  - Constrained simulated annealing converges asymptotically to globally optimal bundles with probability one
  - Stochastic search with iterative deepening finds globally optimal bundles in finite time with high probability
  - Does not require differentiability or continuity of functions
- Lagrange constraints + local objectives

  $\equiv$ Discrete-space Euler-Lagrange conditions for each stage

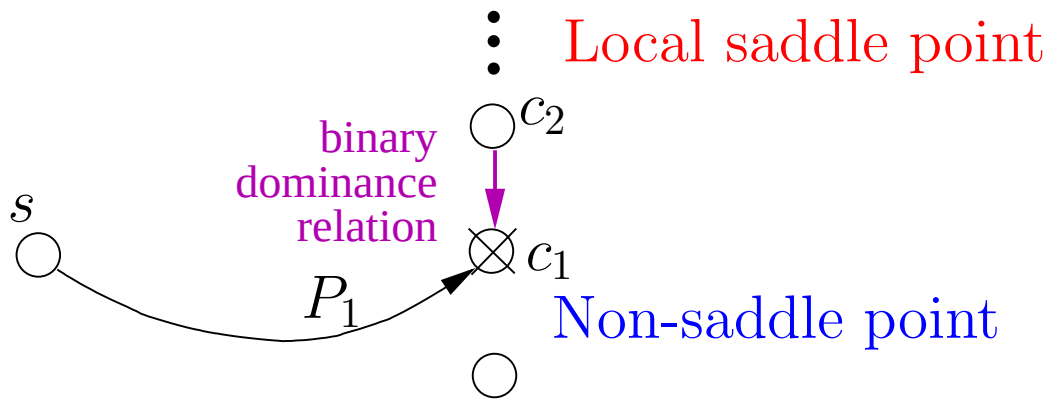
  $\equiv$ Distributed discrete-space saddle-point conditions
- Reduce the problem of finding bundles that satisfy overall SP_{dn}

  $\Longrightarrow$ Finding dominating distributed saddle points SP_{dn} in each stage

Intuitive Meaning Behind Saddle Points



Node Dominance due to Local Saddle Points



Termination of c_1 by
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CX1-PREF Benchmark

- Citizen Explorer-I satellite design and operation planning benchmark
  - Multiple competing preferences to be optimized
  - Problem generator to generate different problem instances

$$perl \quad probgen.pl \quad < random\ seed > \quad < number\ of\ orbits >$$

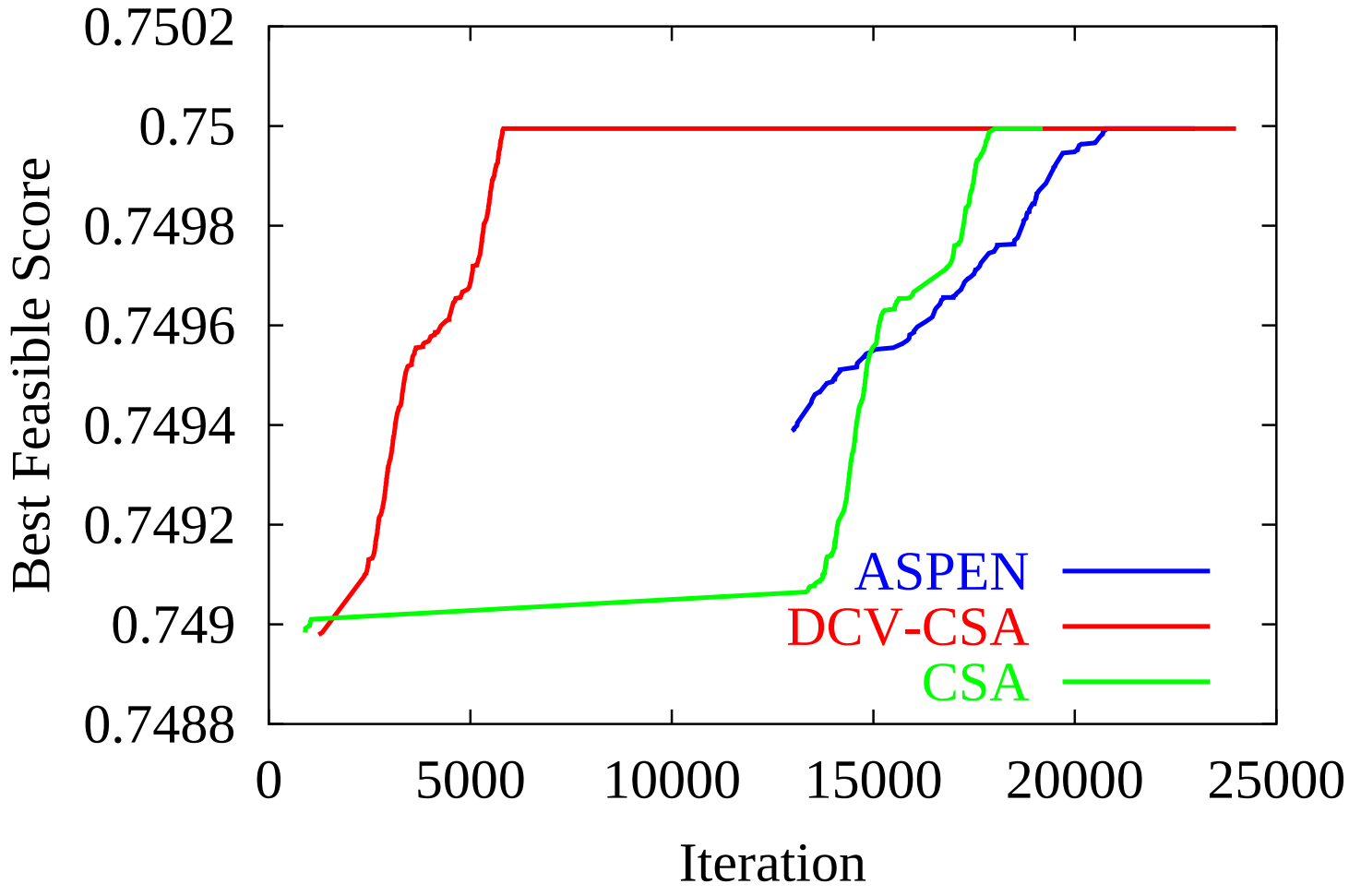
- ASPEN search setting:

  a) Find feasible schedule using *repair*

  b) Optimize score using *optimize* (default 200 iterations)

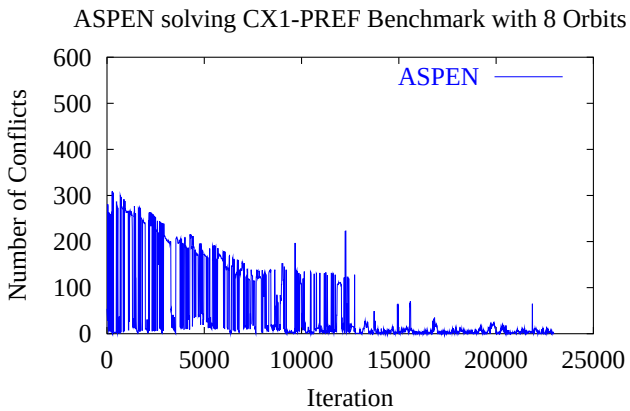
  c) repeat (a) and (b)

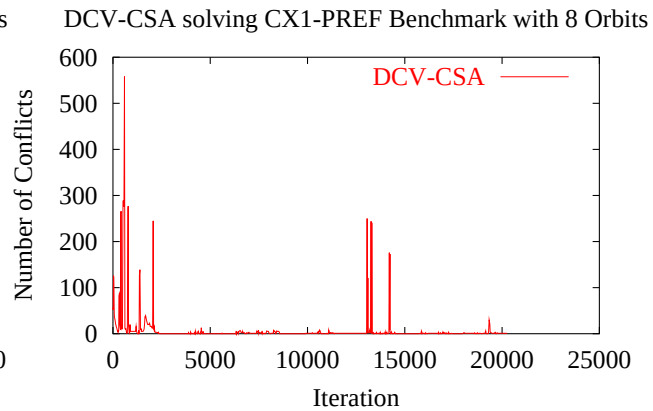
Best Feasible Scores in an 8-Orbit Problem



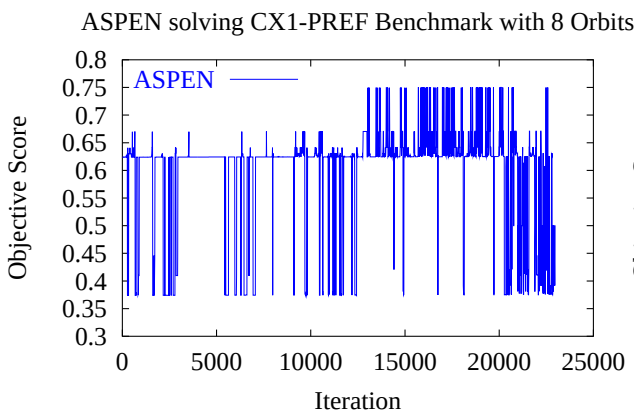
Search Progress in an 8-Orbit Problem

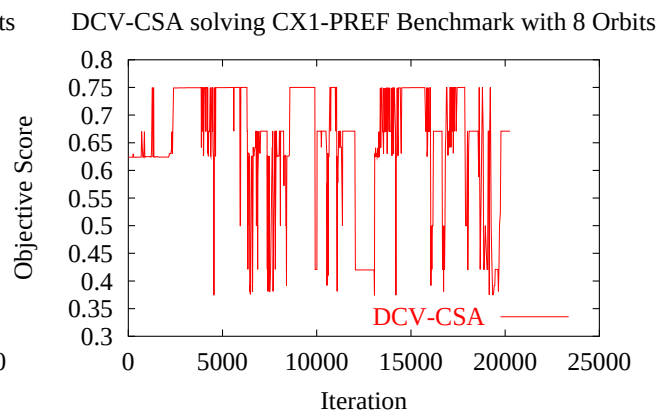
- Number of conflicts vs. Iteration:



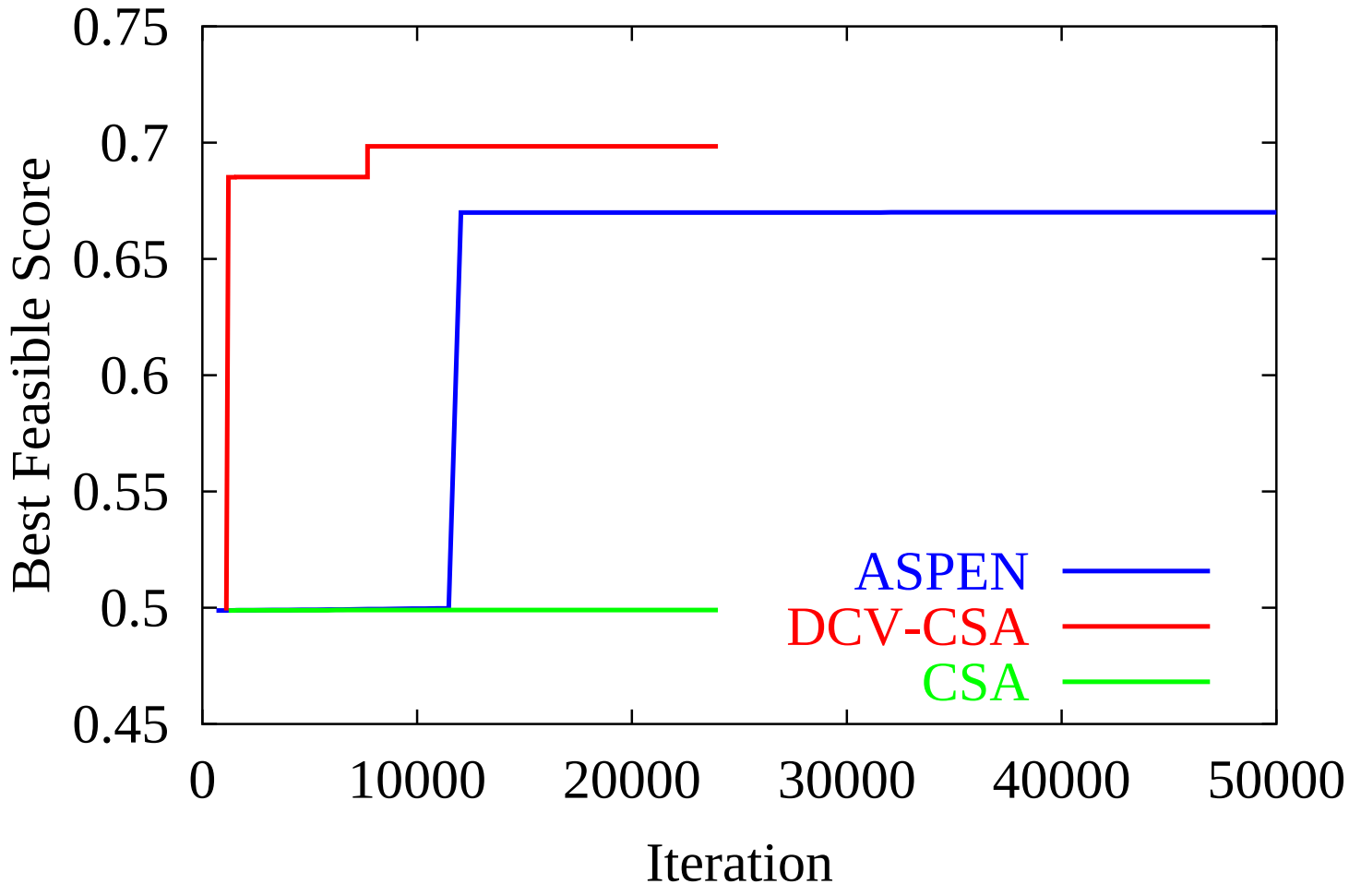


- Score vs. Iteration:



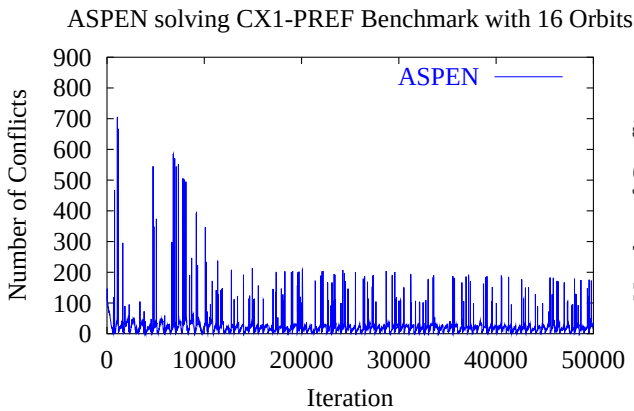


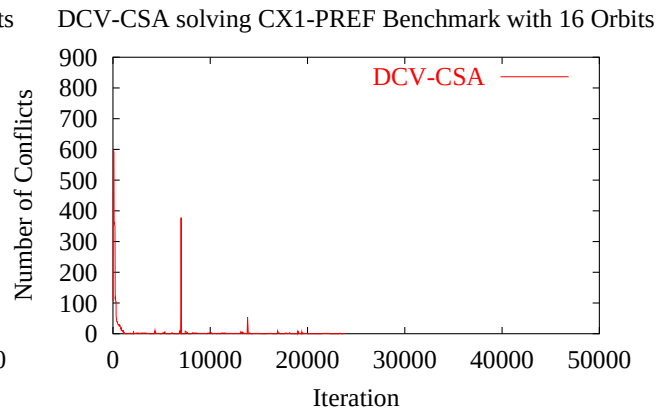
Best Feasible Scores in a 16-Orbit Problem



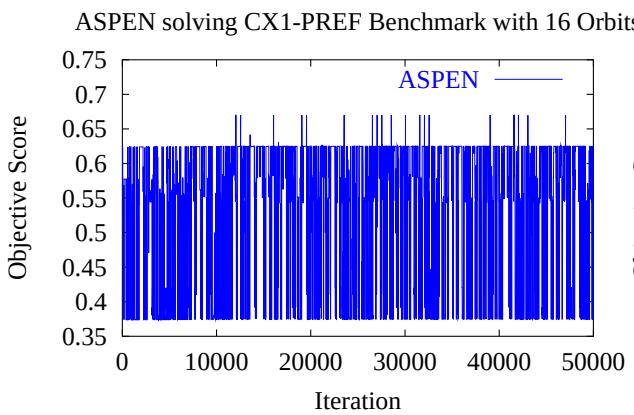
Search Progress in a 16-Orbit Problem

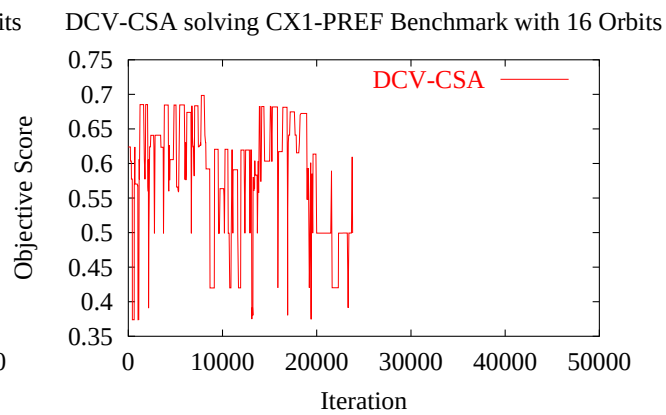
- Number of conflicts vs. Iteration:



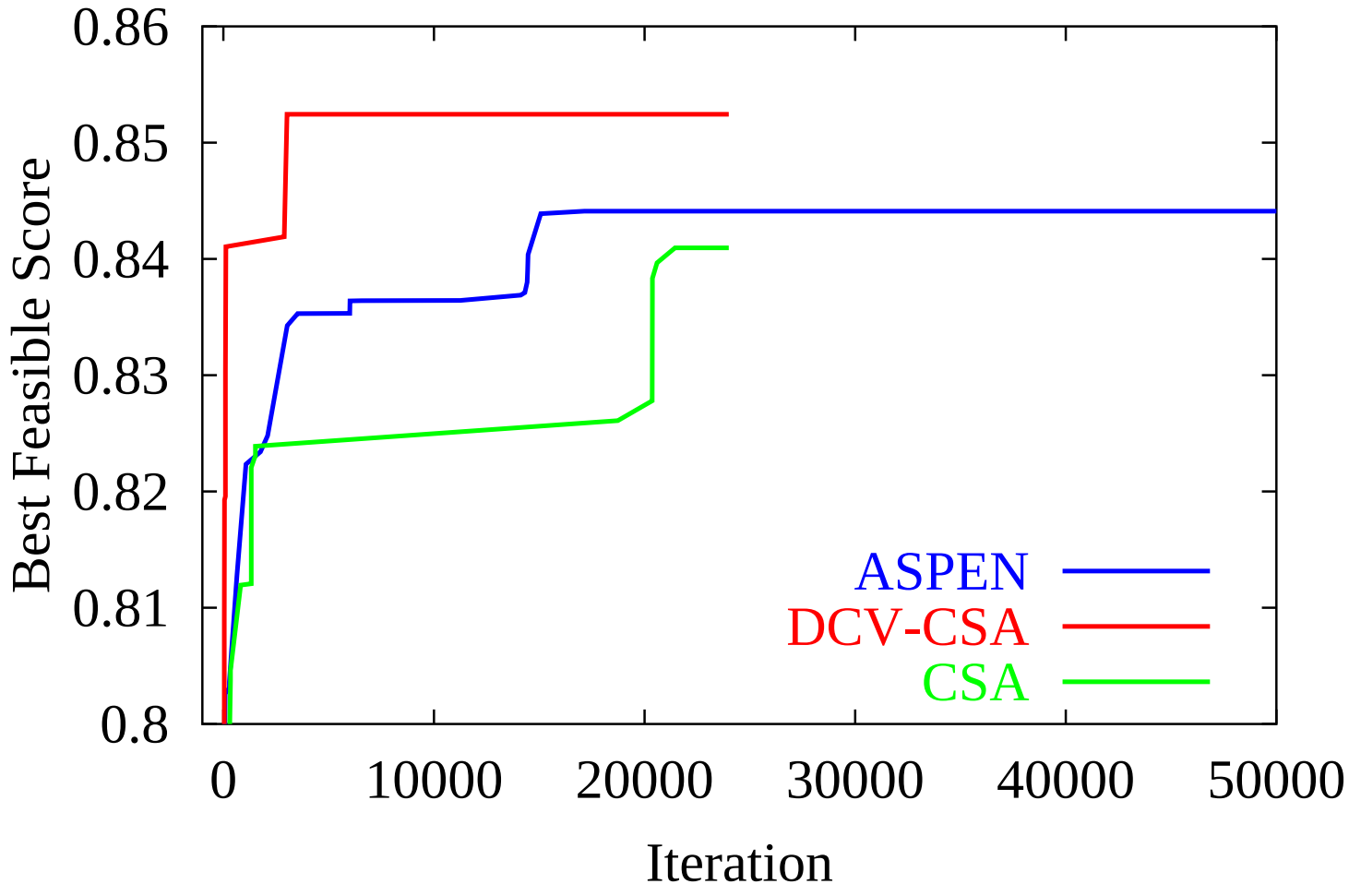


- Score vs. Iteration:



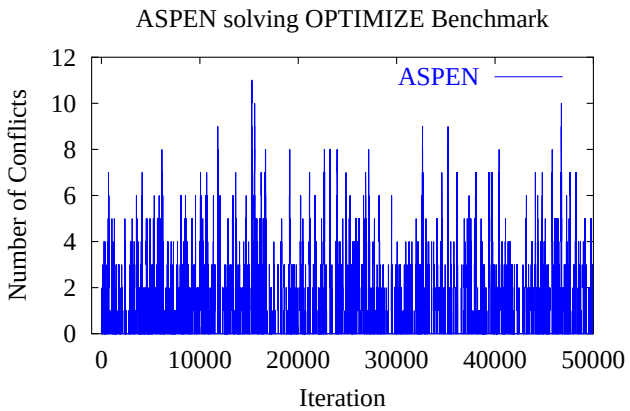


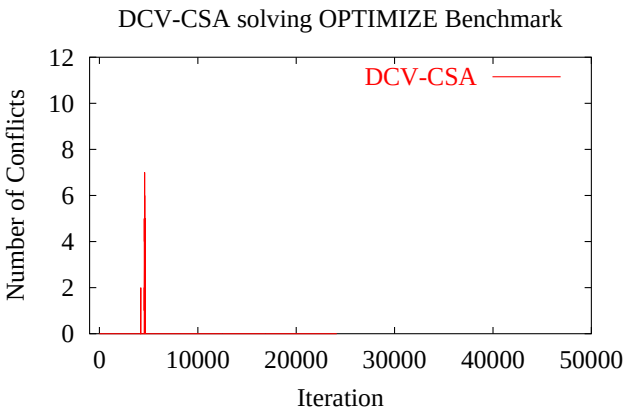
Best Feasible Scores in OPTIMIZE Benchmark



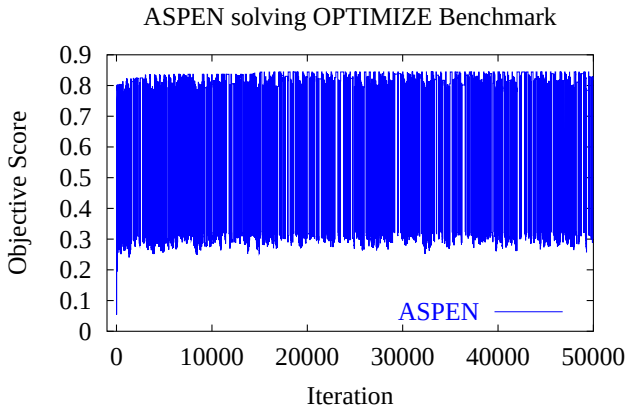
Search Progress in OPTIMIZE Benchmark

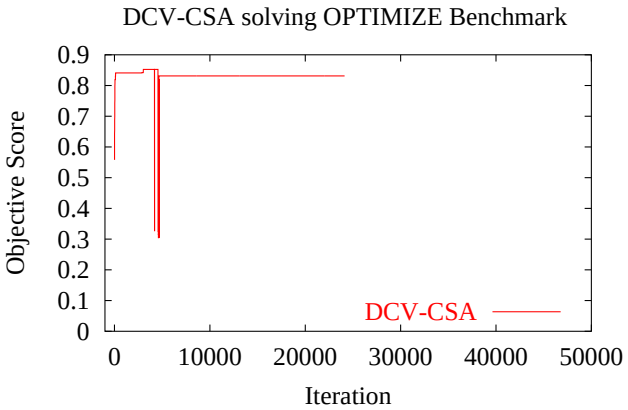
- Number of conflicts vs. Iteration:



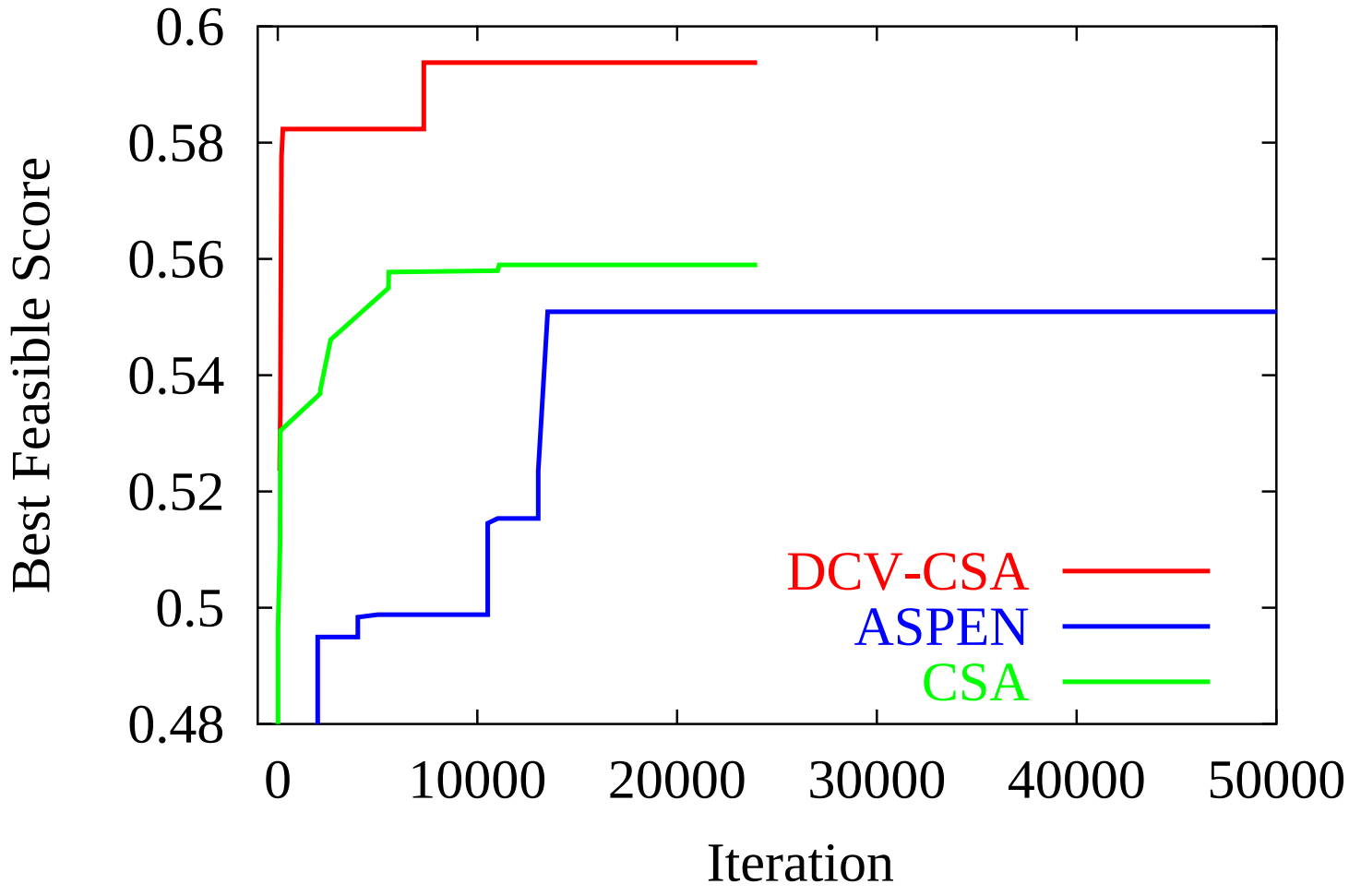


- Score vs. Iteration:



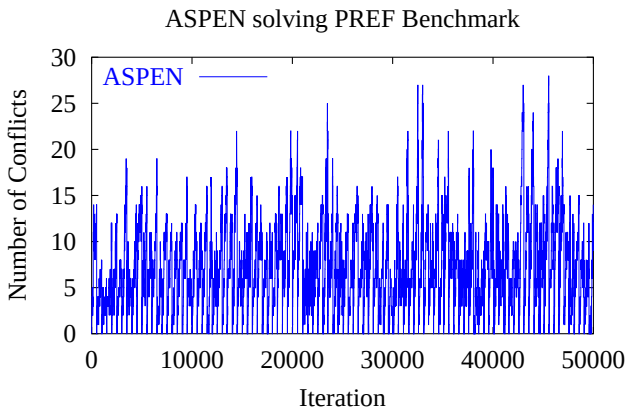


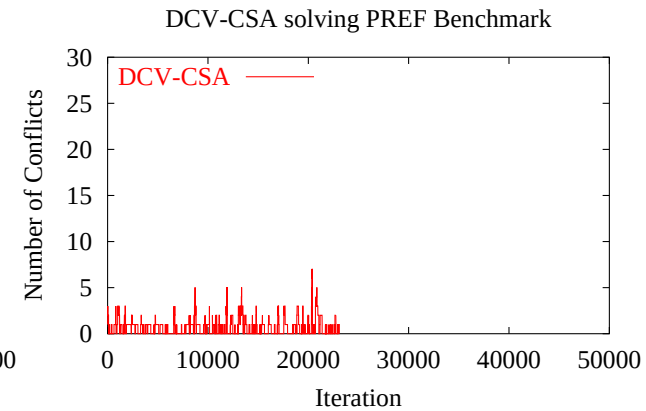
Best Feasible Scores in PREF Benchmark



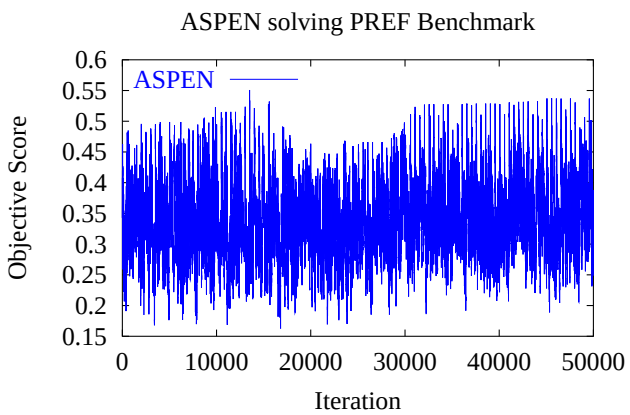
Search Progress in PREF Benchmark

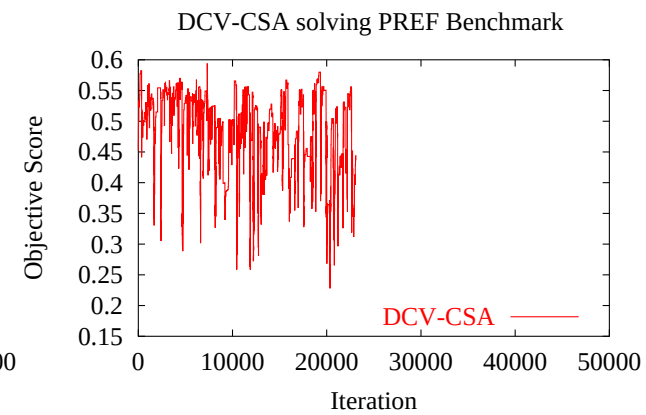
- Number of conflicts vs. Iteration:





- Score vs. Iteration:





Conclusions

- Dynamic optimization with general constraints in discrete time and state
  - Path dominance in dynamic programming cannot be applied
  - Calculus of variations works in continuous state space
- Binary node dominance relations
  - $CLM_{dn} \equiv ELE_{dn} \equiv DSP_{dn}$
  - Rely on theory of Lagrange multipliers for discrete constrained optimization
  - Do not require the differentiability and continuity of functions
  - Significant reduction in the base of the exponential complexity
- Better-quality solutions in shorter time than those of ASPEN